Testing quantum correlations in a confined atomic cloud by scattering fast atoms

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We suggest measuring one-particle density matrix of a trapped ultracold atomic cloud by scattering fast atoms in a pure momentum state off the cloud. The lowest-order probability of the inelastic process, resulting in a pair of outcoming fast atoms for each incoming one, turns out to be given by a Fourier transform of the density matrix. Accordingly, important information about quantum correlations can be deduced directly from the differential scattering cross-section. A possible design of the atomic detector is also discussed.

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Successful advances in achieving collective quantum states in confined clouds of alkaline atoms [1] and in atomic hydrogen [2] make possible studying quantum coherent properties of these systems, as well as the revealing fundamental kinetic processes leading to the formation of the coherence. In Ref. [3], a dynamics of the condensate growth has been observed by means of detecting a density increase usually associated with the condensate formation in the trap. The coherence of the condensate has been demonstrated directly by letting two released condensates form the interference pattern [4].

Mechanism of formation of quantum correlations is a matter of great attention and controversy. An emergence of the condensate due to either lowering temperature or reaching an equilibrium after fast quenching is associated with a formation of the off-diagonal long-range order (ODLRO) [5]. A primary object displaying such an order is the one-particle density matrix (OPDM) $\rho(\mathbf{x}_1, \mathbf{x}_2)$. Typical distances over which these correlations become important are comparable with the interatomic spacing r_a. Consequently, an "early" detection of such emerging correlations is very difficult to achieve by light with the wavelength $\lambda \gg r_a$ usually employed for probing the cloud density. In Ref. [6], it has been suggested that a resonant fluorescence of an external atom identical to those forming the condensate is sensitive to the relative phase of two condensates. However, this method is not suitable for testing distances shorter than λ. Information about short-range density correlations (at distances $< r_a$) can, in principle, be obtained from the absorption of detuned resonant light [7]. The change in local m-body density correlations (the so-called $m!$-effect [8]) can be seen by measuring recombination rates, and this has been already done experimentally for the equilibrium case [9]. However, measurements of the correlation length r_c of the forming ODLRO seem very unlikely to be achieved by these methods.

Thus it is tempting to have a tool which could make possible seeing the OPDM directly without limitations on the accessible distances.

Currently, great efforts are being dedicated to a creation of a controllable source of coherent atoms – atomic LASER (see Ref. [10] and references therein). In Ref. [11], a mechanism of accelerating neutral atoms has been proposed. Hence, it is very likely that a source of fast and coherent atoms will be available soon. In this paper, we suggest a method of detecting the OPDM which relies on inelastic scattering of such atomic beam off the atomic cloud.

We note that methods of scattering of neutrons [12,13] and He atoms [14] off liquid He are well known. In Ref. [12], the Impulse Approximation has been suggested to employ for interpreting the differential cross-section of fast neutrons. In such an approximation, it is possible to relate the momentum transfer distribution to some integral of the population factor in He. However, liquid He is a strongly interacting system where the gas parameter $\xi = na^3$ (n is the density and a is the scattering length) is not small. Accordingly, multiparticle excitations dominate in the final-state channel, which makes a direct measurement of the OPDM impossible. Certain assumptions about the role of the final-state effects should be made [13]. In contrast to this, the gas parameter in the trapped atomic condensates can be as small as $\xi \sim 10^{-5}$. Accordingly, the mean free path is $\approx a/\xi \sim 10^{-2}$ cm, which greatly exceeds the mean particle separation, and can become greater then the cloud size. This implies that a contribution of the multiparticle excitations can be safely neglected as long as a wavelength of the incoming atom is much smaller than the interparticle separation in the atomic cloud. Under these conditions, as we will discuss below, it becomes possible to measure the OPDM directly.

If the mass of the incoming fast atom is comparable to that of the atoms forming a cloud (or, in particular, the fast atom is just identical to the atoms of the cloud), the lowest-order inelastic scattering event is a production of two fast outcoming atoms (the size of atomic cloud is supposed to be small enough to neglect multiple scattering). The quantum-mechanical probability of this process turns out to be proportional to a Fourier transform of the OPDM.

Another process - an elastic scattering of a single fast atom off the cloud - also exists. In this case one has a single fast atom in the final state, in a complete analogy with elastic scattering of light. In principle, the elastic processes might mask the inelastic ones. However, if the momentum of incoming atom, k, is much larger than the typical momentum of the the cloud, $\sim 1/r_c$, then the elastic-scattering angles are small in the parameter $(kr_c)^{-1} \ll 1$. This feature allows one to distinguish between the two processes. One more feature, arising at $kr_c \gg 1$ from the conservation laws, is the fact that the angle between the two created fast atoms is always close to $\pi/2$. This facilitates identification of pairs resulted from one and the same scattering event.

Apart from extremely small uncertainty of the momentum of the incident atom, the method we suggest implies that the total momentum of the outcoming pair can be detected with a sufficient precision. Thus, a reliable atomic detector is required. Therefore, we will also discuss a possible design of such a detector.

Now let us derive an expression for the inelastic cross-section in the lowest order with respect to the two-body interaction. For the sake of simplicity, we will not consider the complexity of all possible scattering channels and will concentrate on a simplest case of spin-polarized bosons, when the incoming fast atom is identical to particles forming the cloud, and its spin polarization is the same as that of the cloud. We emphasize that the validity of the suggested method relies on a possibility to have the incident momentum k obeying the relation

$$\xi^{1/3} \ll ka \ll 1 \ . \tag{1}$$

Then, one can represent the interaction Hamiltonian in a traditional form ($\hbar = 1$)

$$H_{int} \, = \frac{u_0}{2} \int d\mathbf{x}\, \Psi^\dagger(\mathbf{x})\Psi^\dagger(\mathbf{x})\Psi(\mathbf{x})\Psi(\mathbf{x}) \ , \quad u_0 = \frac{4\pi a}{m} \ , \tag{2}$$

with m standing for the atomic mass. The total field Ψ can be subdivided into the low- and the high-energy parts, ψ and ψ', respectively:

$$\Psi = \psi + \psi' \ , \quad \psi' = \sum_{\mathbf{k}} a_{\mathbf{k}}\, \mathrm{e}^{i\mathbf{kx}} \ , \tag{3}$$

where the incident and the scattering states are described in terms of plane waves normalized to unit volume; $a_{\mathbf{k}}$ destroys the high-energy particle with the momentum $\mathbf{k}$. A substitution of Eq. (3) into Eq. (2) and selection of the terms that describe a process involving one incident fast atom carrying momentum $\mathbf{k}$ and two ejected fast atoms carrying momenta $\mathbf{k}_1$ and $\mathbf{k}_2$, as well as its reverse, yield

$$H'_{int} \, = \, u_0 \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}} a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}} \int d\mathbf{x}\, \mathrm{e}^{-i\mathbf{qx}} \psi \, + \, \mathrm{H.c.} \ , \tag{4}$$

where $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}$ is the transferred momentum. The double-differential cross-section for scattering with given energy and momentum transfers, ω and $\mathbf{q}$, in the lowest order with respect to H'_{int} is given by the Golden rule formula

$$W(\mathbf{q},\omega) = \frac{2mu_0^2}{k} \int \frac{d\mathbf{k}_1}{(2\pi)^3}\, \delta(\omega - \omega_{fi}) \iint d\mathbf{x}_1 d\mathbf{x}_2 \int_{-\infty}^{\infty} dt\, \mathrm{e}^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)-i\omega t}\, \rho(\mathbf{x}_1, t; \mathbf{x}_2, 0) \ , \tag{5}$$

where $\rho(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2) = \langle \psi^\dagger(\mathbf{x}_1,t_1)\psi(\mathbf{x}_2,t_2)\rangle$ [Note that unlike the uniformity in time, $\rho(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2) = \rho(\mathbf{x}_1,t_1 - t_2;\mathbf{x}_2,0)$, the space uniformity cannot be, in general, assumed as long as a trapping potential exists.]; ω_{fi} is the difference of the kinetic energies of the fast atoms in the final and initial states:

$$\omega_{fi} = \frac{1}{m}\left[\frac{q^2}{2} + \mathbf{qk} + k_1^2 - (\mathbf{q}+\mathbf{k})\mathbf{k}_1\right] \ , \tag{6}$$

with $\mathbf{k}_1$ being the momentum of one of the outcoming atoms while the momentum of the second one, $\mathbf{k}_2$, is fixed by the relation $\mathbf{k}_2 = \mathbf{q} + \mathbf{k} - \mathbf{k}_1$.

The integration over $\mathbf{k}_1$ in Eq. (5) can be carried out explicitly. However, first we notice that the requirement of large k means that the values of q and ω, which are effectively selected by the correlator $\rho(\mathbf{x}_1, t; \mathbf{x}_2, 0)$ in the right-hand side of Eq. (5), satisfy the conditions $q \ll k$ and $|\omega| \ll k^2/m$. This immediately leads to the approximation $\delta(\omega - \omega_{fi}) \approx m\, \delta(k_1^2 - \mathbf{kk}_1)$, which yields

$$W(\mathbf{q},\omega) = 4a^2 \iint d\mathbf{x}_1 d\mathbf{x}_2 \int_{-\infty}^{\infty} dt\, \mathrm{e}^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)-i\omega t}\, \rho(\mathbf{x}_1, t; \mathbf{x}_2, 0) \ . \tag{7}$$

We thus see that the double-differential cross-section $W(\mathbf{q},\omega)$ is almost directly related to the dynamic correlator $\rho(\mathbf{x}_1,t;\mathbf{x}_2,0)$ which, as is known, contains rather rich information about the system, including, for one thing, the elementary excitation spectrum. Confining ourselves to the static correlations, described by OPDM $\rho(\mathbf{x}_1,\mathbf{x}_2) = \rho(\mathbf{x}_1,0;\mathbf{x}_2,0)$, we arrive at even more simple relation

$$W(\mathbf{q}) = 8\pi a^2 \iint d\mathbf{x}_1 d\mathbf{x}_2\, \mathrm{e}^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}\rho(\mathbf{x}_1,\mathbf{x}_2) \tag{8}$$

in terms of the differential cross-section $W(\mathbf{q}) = \int d\omega\, W(\mathbf{q},\omega)$.

Consider the structure of $W(\mathbf{q})$ in the most characteristic cases. In the case of *pure Bose-Einstein condensate*, when $\rho(\mathbf{x}_1,\mathbf{x}_2) = \Phi^*(\mathbf{x}_1)\Phi(\mathbf{x}_2)$ [$\Phi(\mathbf{x})$ is the condensate wave-function],

$$W(\mathbf{q}) \;=\; 8\pi a^2|\Phi_{\mathbf{q}}|^2\;, \quad \Phi_{\mathbf{q}} = \int d\mathbf{x}\,\mathrm{e}^{-i\mathbf{q}\mathbf{x}}\,\Phi(\mathbf{x})\;. \tag{9}$$

In particular, if $\Phi_{\mathbf{q}}$ is real, Eqs. (9) can be reversed, and correspondingly $\Phi(\mathbf{x})$ can be restored from $W(\mathbf{q})$.

Another instructive case is the *axially symmetric quantum vortex.* A presence of a single vortex in a center of the axially symmetric condensate drastically changes the scattering pattern. Indeed, in this situation, $\Phi = \exp(i\theta)\sqrt{n(r,z)}$, where θ is the axial angle and $n(r,z)$ stands for the axially symmetric condensate density as a function of the distances r, z perpendicular to the axis and along the axis, respectively. Accordingly, Eq. (9) shows that $W(\mathbf{q}) = 0$ for $\mathbf{q}$ directed along the vortex line. The differential cross-section becomes finite as long as there is a component $\mathbf{q}_\perp$ of $\mathbf{q}$ perpendicular to the axis or the vortex displaces from the condensate center. In the first case, $W(\mathbf{q}) \sim q_\perp^2$ for $q_\perp \to 0$.

Quite similar to the quantum vortex is the case of the *supercurrent state* of a toroidal Bose condensate. The suppression of $W(\mathbf{q})$ for $\mathbf{q}$ perpendicular to the plane of the torus allows distinguishing the supercurrent state from the currentless genuine ground state.

In the absence of condensate, or for the above-the-condensate part of OPDM, one normally deals with the so-called *quasi-homogeneous regime*, when typical inverse momentum of particles is much less then the scale of density variation. In this case, it is reasonable to introduce the variables $\mathbf{r} = \mathbf{x}_2 - \mathbf{x}_1$ and $\mathbf{R} = (\mathbf{x}_1 + \mathbf{x}_2)/2$, and to represent $W(\mathbf{q})$ as

$$W(\mathbf{q}) = 8\pi a^2 \int d\mathbf{R}\,\rho_{\mathbf{q}}(\mathbf{R})\;, \quad \rho_{\mathbf{q}}(\mathbf{R}) = \int d\mathbf{x}\,\mathrm{e}^{-i\mathbf{q}\mathbf{x}}\,\rho(\mathbf{x},\mathbf{R})\;, \tag{10}$$

since in the quasi-homogeneous regime, OPDM in the Wigner representation, $\rho_{\mathbf{q}}(\mathbf{R})$, has a semiclassical meaning of local (at the point $\mathbf{R}$) distribution of the particle momentum $\mathbf{q}$. We thus see that in this case $W(\mathbf{q})$ yields spatially averaged momentum distribution. Moreover, without contradiction with the uncertainty principle, this averaging can be partially (totally) removed by collimating the incident (both the incident and outcoming) beams. Even without removing the averaging, $W(\mathbf{q})$ contains valuable information about long-range correlations in the system, since the the averaging does not affect the order-of-magnitude value of the correlation radius r_c (equal to the inverse typical momentum k_c). If the system contains both condensate and quasi-homogeneous above-the-condensate fraction, the cross-section $W(\mathbf{q})$ is given by a combination of Eqs. (9) and (10).

Now let us discuss one possible method of detecting a total momentum of an outcoming pair of atoms. There is a significant requirement for such a detector: an uncertainty of detecting the momentum transfer must be less than the inverse correlation length r_c in the cloud. This imposes a limitation on the detector size R_D. Indeed, let us suppose that cells sensitive to the arrival of the scattered atoms are located on a sphere of the radius R_D, and the atomic cloud of the size $R \ll R_D$ is at the center of this sphere. Then, the uncertainty in the scattering angle is R/R_D. This produces the uncertainty in the momentum $\sim kR/R_D$. This uncertainty must be much less than $1/r_c$, or

$$R_D \;\gg\; kr_c\,R\;. \tag{11}$$

It can be shown that the absolute values of the scattered momenta measured by the time of flight are also subjected to the same uncertainty. Thus, the condition (11) determines a precision of the time-of-flight measurements as well.

Our method implies that two outcoming atoms produced by one incoming atom can be identified. As mentioned above, this identification can be done due to the $\approx \pi/2$ angle between the scattered atoms. However, if the incoming coherent beam produces too many scattered pairs, the erroneous identification is very likely. Let us derive a limitation on the number N_s of the scattered pairs per typical time of flight, which would keep the momentum uncertainty less than $1/r_c$. The scattering events under consideration should occur with approximately equal probability at any angle inside the 4π solid angle, so that the angular area occupied by a single event is $4\pi/N_s$. It should be much less than

$2\pi/r_c k$ which determines the angular area of the strip where the second atom of a pair can be found, the direction of the first being fixed. Thus,

$$N_s \ll 2kr_c\,. \tag{12}$$

If a typical speed of the fast atoms is 10cm/s, and $r_c \approx 10^{-3}$ cm, $R \approx 10^{-2}$ cm, the conditions (11) and (12) yield $R_D \gg 10$ cm and $N_s \ll 10^3$. Accordingly, a typical time of flight will be longer than 1 s.

Now let us discuss a mechanism of detection of a single neutral atom. We suggest employing resonant atomic fluorescence in the evanescent field of light propagating inside a waveguide. Specifically, light sensitive cells are mounted on a side of a long waveguide so that light propagating inside the waveguide does not excite these cells. If holes are made through the cells and the waveguide, the light will remain confined as long as a diameter of the hole is much smaller than the wavelength of light. However, a single atom may enter the hole and feel the resonant field inside the hole, or in the close vicinity of the entrance to it, due to the evanescent field of the light. Consequently, the atom will reemit resonantly one or several photons. These photons can then be detected by the nearest cell indicating an arrival of an atom at a specific location at the detector surface. Accordingly, a scattering angle of the coming atom can be identified. The velocity of the atom can be deduced from the time of flight.

A probability P_D for atom to penetrate into a hole is simply the area A_h of the holes per unit area of the detector exposed to the atomic flux. In order to detect an atom in the hole, the atom must reemit at least few photon which are captured by the cell mounted outside the waveguide. Before we estimate a number N_D of the reemitted photons which can be detected, we note that the photons remitted by the atom, which has penetrated deeply inside the waveguide, should remain confined inside it, and therefore they will not be detected by the cell. Only those photons, which are reemitted by the coming atom while being close to the hole entrance, will be scattered almost isotropically and can be absorbed by the cell. Taking into account that the penetration length of the evanescent field is comparable with the wavelength λ of light, one can find the time $t_D \approx \lambda/v$ during which the atom moving at speed v is subjected to the evanescent field and reemits light isotropically. Then, the number of photons reemitted during this time is $N_D \approx \gamma t_D = \gamma\lambda/v$, where γ stands for the natural width of the line. Choosing typical values $\lambda = 700$nm, $\gamma \approx 10^7$ Hz and $v = 10$ cm/s one finds $N_{ph} = 70$. Some geometrical factor of the order of one should reduce the number of photons reaching the cell. Quantum efficiency of photodetectors can be easily achieved to be 10%-20%. This implies that once an atom entered a hole, it will be detected with high certainty. Therefore, a probability to detect an atom at a given position is just P_D. In other words, if a diameter of each hole is 100 nm and two closest holes are 300 nm apart, the probability is $P_D \approx 0.1$. Thus, in order to achieve a resolution of at least .1 of the typical momentum region in the scattering intensity, one needs at least 10 scattering events for each component of the momentum. This corresponds to 10 atoms ejected from the cloud (plus 10 fast incident atoms). Accordingly, for the 3D geometry, it translates into $10^3/P_D \approx 10^4$ atoms in total. In a typical condensate of 10^6 atoms [1], such a bombardment by fast atoms will result in a depletion of the cloud by only 1%. Evidently, this method is not appropriate for clouds containing less than 10^5 atoms. In the effective 2D or 1D geometries, the same resolution requires much less numbers of the scattering events. Specifically, 10^3 and 10^2 of them for the 2D and 1D geometries, respectively, will satisfy the above criteria of resolution.

In conclusion, we have suggested a method of scattering of fast atoms in a pure enough momentum state off a trapped atomic cloud in order to test directly one-particle density matrix of this cloud. The differential cross-section of the inelastic process, when one incoming fast atom produces two fast ones, allows measuring the correlation length of the local off-diagonal order. This gives, in particular, a powerful tool for testing different scenarios of formation of the off-diagonal long-range order in the traps. This method can also be employed for detecting quantum vortices and supercurrent states, as well as the effect of quantum depletion of the condensate. The main principles and a design of the detector of scattered atoms are suggested.

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